YEAR 10 | QUESTIONS ONLY

# Algebra: from identities to proofs

LESSON 1

## Use identities to see more

**B1.1** Factor $(2a+b)^2-(a-2b)^2$.

**B1.2** Let $x+y=7$, $xy=10$. Find $x^2+y^2$ and $x^3+y^3$ without finding $x$ and $y$ separately.

**B1.3** If $x+y=1$, find $x^3+y^3+3xy$.

**B1.4** Real numbers $a,b,c$ satisfy

$$(a+b)^2+(b+c)^2+(a+c)^2=(a+b+c)^2.$$

Find all possible triples $(a,b,c)$.

**B1.5** Real numbers $a,b,c,d$ satisfy $a+b=c+d$ and $a^2+b^2=c^2+d^2$. Must $a^3+b^3=c^3+d^3$? Prove your answer.

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LESSON 2

## Make a useful product appear

**B2.1** Factor $ab+ac+bd+cd$.

**B2.2** Factor $6x^3y-9x^2y^2+3xy^3$.

**B2.3** Factor $x^4+x^2+1$.

**B2.4** Prove that for every positive integer $n$, the number $n^4+4n^2+3$ is composite.

**B2.5** Prove that $2020\cdot 2022\cdot 2024\cdot 2026+16$ is a perfect square.

# Algebra: from identities to proofs

LESSON 3

## Find all the possibilities

**B3.1** Find all ordered pairs of positive integers $(x, y)$ satisfying $xy = 12$.

**B3.2** Find all integer solutions of $x^2 - y^2 = 15$.

**B3.3** Find all integer solutions of $x^2 - 4y^2 = 12$.

**B3.4** Find all integers $a$ for which $x^2 - ax + 12 = 0$ has two integer roots.

**B3.5** Find every positive integer $n$ for which $n^2 + 2n + 12$ is the product of two consecutive positive integers.

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LESSON 4

## Squares give bounds

**B4.1** Prove that for all real $a, b$, $a^2 + b^2 \geq 2ab$.

**B4.2** Find the least value of $x^2 - 8x + y^2 + 2y + 20$ for real $x, y$.

**B4.3** Let $x, y > 0$ and $x + y = 10$. Prove that $xy \leq 25$.

**B4.4** Prove that $a^2 + b^2 + c^2 \geq ab + bc + ca$ for all real $a, b, c$. State when equality holds.

**B4.5** Find the greatest possible value of

$$20x - 4y + 6z - 2x^2 - 4y^2 - 3z^2 - 2$$

for real numbers $x, y, z$.