YEARS 5-7 | QUESTIONS ONLY

Logic and problem solving

LESSON 1

Examples and logical conclusions

A1.1 Can three positive integers have their sum equal to their product? Give an example or prove that it is impossible.

A1.2 Kolya's mother says, "Every champion does well at school." Kolya replies, "I do well at school, so I am a champion." Does his conclusion follow? Explain.

A1.3 A positive integer n is divisible by both positive integers a and b. Can n fail to be divisible by ab? Give an example or a proof.

A1.4 Some residents of a town have beautiful handwriting. No poet has beautiful handwriting, and every boxer is a poet. A visitor claims that every resident is a boxer or a poet. Can the claim be true? Explain.

A1.5 A zoo with both hippos and rhinos has no giraffes. Every zoo has at least a rhino or a hippo. Any zoo with hippos and giraffes also has rhinos. A particular zoo has a giraffe. Must it have a rhino? Can it have a hippo?

LESSON 2

What cannot change?

A2.1 The number 4 is written on a board. In one move, one may add 2. Can 99 be obtained?

A2.2 There are piles of 3, 5, and 7 stones. In one move, one stone may be moved from one pile to another. Can the piles become 4, 6, and 10?

A2.3 There are 9 coins heads up. In one move, exactly two coins are flipped. Can all coins become tails up?

A2.4 Two opposite corner squares are removed from an 8×8 chessboard. Can the remaining squares be tiled by 1×2 dominoes, with each domino covering two squares sharing a side?

A2.5 Thirty-three cups are upside down. In one move, you turn over exactly (a) two cups, (b) six cups, or (c) five cups. Treat the three rules as separate games. Under each rule, can you make every cup stand upright? Justify each answer.

Logic and problem solving

LESSON 3

A match is unavoidable

A3.1 Prove that among 13 people, two were born in the same month.

A3.2 There are 102 rabbits in 10 cages. Prove that some cage contains at least 11 rabbits.

A3.3 A drawer contains socks of two colors. Prove that among any 5 socks taken out, 3 have the same color.

A3.4 From $1, \dots, 10$, 6 numbers are chosen. Prove that two chosen numbers have sum 11.

A3.5 There are 90 rabbits in 14 cages. Prove that two cages contain the same number of rabbits. Empty cages are allowed.

LESSON 4

A strategy that always works

A4.1 There are 14 stones in a pile. In one move, a player may take 1 or 2 stones. Whoever takes the last stone wins. Who wins with perfect play?

A4.2 Two players take turns removing one, two or three stones from a pile of 30. A player with no legal move loses. Which player can force a win, and how?

A4.3 There are two piles of 10 stones each. In one move, a player may take any positive number of stones from one pile. The last move wins. Prove that the second player wins.

A4.4 Three piles contain 10, 15 and 20 stones. Players take turns splitting one pile containing more than one stone into two non-empty piles. A player who cannot move loses. Who wins? Prove that your answer does not depend on the choices made.

A4.5 A row contains one banana on each plate. Players take turns taking a banana from one plate, or taking the bananas from two adjacent plates that both still contain a banana. Empty plates stay in place. The last player to take a banana wins. Find a winning strategy for (a) 20 plates and (b) 21 plates.