

Visual geometry

Year 8-9 | Four-lesson problem ladder

Use a separate sheet for your working. Explain your thinking with words, calculations and sketches. Figures are schematic; use stated lengths and properties, not measurements from the page.

This pack contains 24 problems, arranged in four sets of six. Each set starts with accessible tasks and ends with a stretch task. Your teacher will choose the core questions for each meeting. Separate hints and explained solutions are available later. **Tools to keep nearby**

- Rectangle: $P = 2(L + W)$, $A = LW$. Triangle: $A = \frac{1}{2}bh$ with a perpendicular height. Rhombus and parallelogram: $A = bh$.
- A closed rectilinear path has equal total travel in opposite directions. Remove inside edges when finding a union's outside perimeter.
- **Similar figures:** length ratio k gives area ratio k^2 . Joining the side midpoints of a rectangle produces a parallelogram of half its area.
- **Pythagoras:** in a right triangle, $a^2 + b^2 = c^2$, with c the hypotenuse.
- Triangle angles sum to 180° ; angles around a point sum to 360° . Equal sides in a triangle have equal opposite angles. An equilateral triangle has three 60° angles.
- A regular n -gon has exterior angle $360^\circ/n$ and interior angle $180^\circ - 360^\circ/n$. An n -gon's interior angles sum to $(n - 2)180^\circ$.
- Cuboid volume: abc . Prism volume: base area times prism length. Surface area counts only exposed faces. A painted integer cuboid has $(a - 2)(b - 2)(c - 2)$ unpainted unit cubes when all dimensions exceed 2.
- A disc of radius r has area πr^2 ; four equal quarter-discs make one full disc.

What makes a complete solution?

State why each step follows. For “all possible”, list cases and explain why they cover everything. For a construction, check all requirements. A drawing suggests a claim; it does not prove it. **The four steps**

1. **Recover hidden information.** Balanced boundaries, angle sums, midpoint areas and overlap edges.
2. **Scale, compare and count.** Area ratios, Pythagoras, equal partitions and systematic grid counts.
3. **Solid geometry and structure.** Cube orientations, surface area, nets, integer cases and growth.
4. **Cases and proofs.** Classify partitions, find repeating blocks, and write complete geometric arguments.

Lesson 1 Recover hidden information

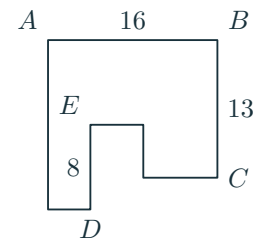
Balanced boundaries, angle sums, midpoint areas and overlap edges.

Suggested route: first four questions as the core; last two as extensions. Your teacher may adjust this.

B01 A spiral boundary

Supplied extract 7.1.9 | Collection exercise; no contest is cited

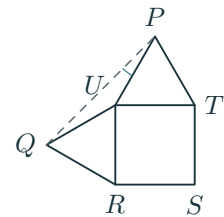
The polygon has perpendicular consecutive sides. Given $AB = 16$, $BC = 13$ and $DE = 8$, find its perimeter.



B02 A square and two triangles

Britain | UKMT Junior Mathematical Challenge 2022, Q6

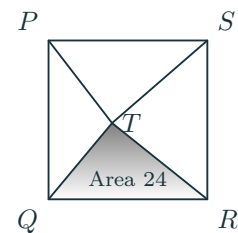
$RSTU$ is a square. Equilateral triangles PUT and QRU are drawn outside it, as shown. Find $\angle QPU$.



B03 Two opposite triangles

Britain | UKMT Junior Mathematical Challenge 2024, Q17

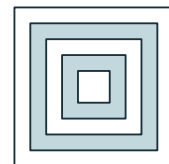
Square $PQRS$ has area 100 cm^2 . Point T lies inside it, and triangle QRT has area 24 cm^2 . Find the area of triangle PTS .



B04 Nested squares

Britain | UKMT Junior Mathematical Challenge 2022, Q13

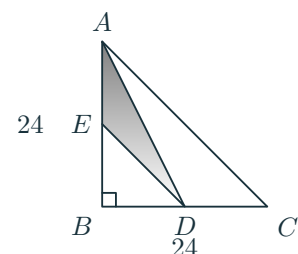
The five nested squares have side lengths 1, 2, 3, 4, 5. The bands between sides 1 and 2, and between sides 3 and 4, are shaded. What percentage of the largest square is shaded?



B05 Midpoints in a right triangle

Canada | CEMC Gauss 2023, Grade 7, Q17

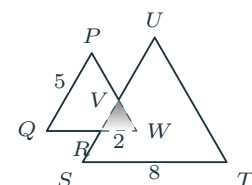
Triangle ABC is right-angled at B , with $AB = BC = 24 \text{ cm}$. Points D and E are the midpoints of BC and AB . Find the area of triangle AED .



B06 Overlapping equilateral triangles

Britain | UKMT Junior Mathematical Challenge 2022, Q16

Equilateral triangles PQW and STU , of sides 5 cm and 8 cm, overlap in equilateral triangle RVW of side 2 cm. Find the perimeter of the outer polygon $PQRSTUV$.



Lesson 2 Scale, compare and count

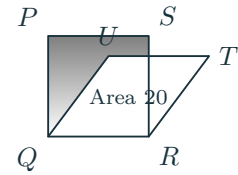
Area ratios, Pythagoras, equal partitions and systematic grid counts.

Suggested route: first four questions as the core; last two as extensions. Your teacher may adjust this.

B07 Square minus rhombus

Britain | UKMT Junior Mathematical Olympiad 2023, A9

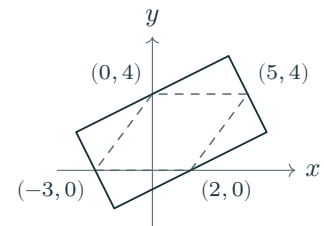
Square $PQRS$ has area 25 cm^2 . Rhombus $QRTU$ has area 20 cm^2 and shares side QR . Find the shaded area of the square outside the rhombus, in the arrangement shown.



B08 A rectangle from its side midpoints

USA | MAA AMC 8, 2022, Q18

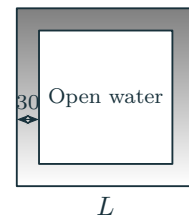
The four side midpoints of a rectangle are $(-3, 0)$, $(2, 0)$, $(5, 4)$ and $(0, 4)$. Find its area.



B09 A freezing pond

Supplied extract 7.3.2 | Formula of Unity / Third Millennium 2018, Grade 5, Q2

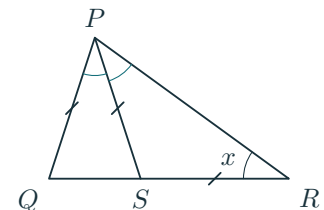
A pond is square. After day k , all water at distance at most $10k$ metres from the nearest shore is frozen. After three days the open-water area has decreased by 19%. After how many days is the pond completely frozen?



B10 An angle bisector with equal lengths

Britain | UKMT Junior Mathematical Challenge 2024, Q13

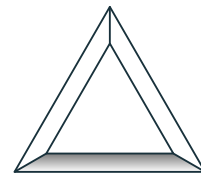
In triangle PQR , point S lies on QR , $\angle QPS = \angle SPR$, and $PQ = PS = SR$. Find $x = \angle PRQ$.



B11 A triangular frame

USA | MAA AMC 8, 2023, Q19

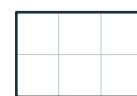
An equilateral triangle contains a smaller parallel-sided equilateral triangle whose side is $2/3$ of the outer side. The region between them is split into three congruent trapezoids, as shown. Find the ratio of one trapezoid area to the inner triangle area.



B12 Toothpicks inside a grid

Canada | CEMC Gauss 2024, Grade 7, Q22

A 2×3 grid uses 17 toothpicks: 10 on its outer boundary and 7 inside. For a 20×24 grid, what percentage of all toothpicks are inside? Round to the nearest whole percent.



Lesson 3 Solid geometry and structure

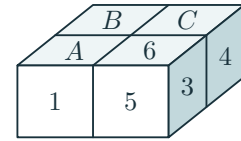
Cube orientations, surface area, nets, integer cases and growth.

Suggested route: first four questions as the core; last two as extensions. Your teacher may adjust this.

B13 Four glued dice

Supplied extract 7.2.4 | All-Russian Olympiad 2023, school round, Grade 5, Q6

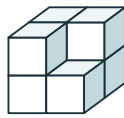
Four identically labelled dice, with faces 1 to 6 and opposite faces summing to 7, form the pictured one-layer 2×2 block. On every contact, the two touching faces have equal numbers. The shown numerals represent pip counts. Find the top-face numbers A, B, C .



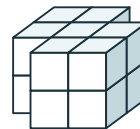
B14 Interlocking solids

Britain | UKMT Junior Mathematical Olympiad 2023, A7

Remove a corner unit cube from a $2 \times 2 \times 2$ cm cube. Two copies of the resulting solid, with one inverted, interlock to form the combined solid shown. Find its total surface area.



One solid

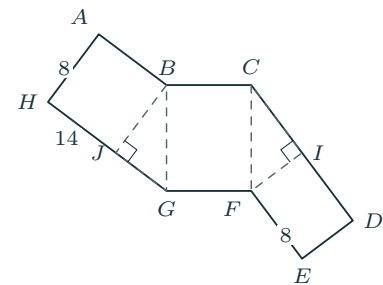


Combined solid

B15 Fold a triangular prism

USA | MAA AMC 8, 2022, Q24

The polygon $ABCDEFGH$ is a net of a triangular prism; dashed segments are fold lines. Given $AH = EF = 8$ and $GH = 14$, find the prism volume.



B16 Painted cuboids: every possibility

Canada | CEMC Gauss 2024, Grade 7, Q23

A cuboid with whole-number side lengths is painted on all six faces, then cut into unit cubes. Exactly 50 small cubes have no paint. Find the arithmetic mean of all possible original volumes.

B17 Nine digits on a cuboid

Canada | CEMC Gauss 2023, Grade 7, Q23

The three side lengths of a cuboid are distinct one-digit positive integers. Its three different face areas are two-digit integers. When all six numbers are written down, the digits 1 through 9 each occur exactly once. Find its surface area.

B18 A growing tree

Britain | UKMT Junior Mathematical Olympiad 2024, A8

Diagram 1 is one vertex. Diagram 2 adds two edges of length 1 cm, each ending at a new vertex. At every later stage, two new edges are added to every existing vertex, each half as long as the edges added at the previous stage. Find the total edge length in diagram 5.



Diagram 1



Diagram 2

Lesson 4 Cases and proofs

Classify partitions, find repeating blocks, and write complete geometric arguments.

Suggested route: first four questions as the core; last two as extensions. Your teacher may adjust this.

B19 Three rectangles with equal perimeters

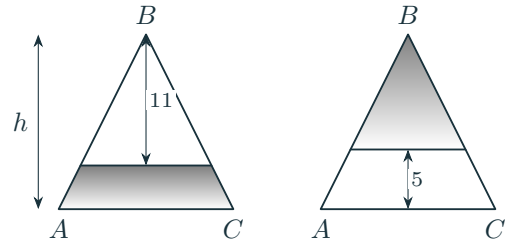
Supplied extract 7.3.6 | Formula of Unity / Third Millennium 2020, Grade 5 Q4 / Grade 9 Q1

A square of side 12 cm is cut into three rectangles with equal perimeters. Find every possible value of the common perimeter, and justify that your list is complete.

B20 Equal shaded areas

USA | MAA AMC 8, 2023, Q24

In isosceles triangle ABC , $AB = BC$ and the height to AC is h . Two different lines parallel to AC create the shaded regions shown. The left unshaded top triangle has height 11; the right unshaded bottom trapezoid has height 5. The shaded areas are equal. Find h .



B21 Both diagonals of a large grid

Supplied extract 7.3.4 | Formula of Unity / Third Millennium 2021, Grade 5, Q3

A rectangle consists of 303 columns and 202 rows of unit cells. Both diagonals are drawn. Colour every cell whose interior is entered by at least one diagonal; touching only a corner does not count. How many cells are coloured?

B22 Island area and coastal water

Supplied extract 7.3.8 | Formula of Unity / Third Millennium 2015, Grade 5, Q5

In a flat world, two islands are rectangles. An island's coastal waters are the sea points within 50 km of its shore; other land is farther than 50 km away. Can the first island have greater land area, while the second has greater coastal-water area? Give an example and justify it.

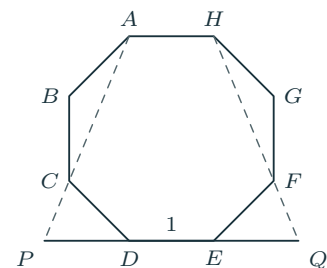


Coastal water (schematic)

B23 An octagon and two extended lines

Britain | UKMT Junior Mathematical Olympiad 2024, B5

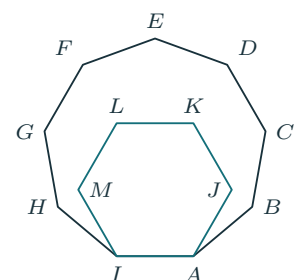
Regular octagon $ABCDEFGH$ has side 1 cm. Lines AC and HF meet line DE at P and Q , respectively. Find PQ , with proof.



B24 A nonagon meets a hexagon

Britain | UKMT Junior Mathematical Olympiad 2023, B4 | Optional proof challenge

Regular nonagon $ABCDEFGHI$ and regular hexagon $AJKLM$ share side IA , with the hexagon inside the nonagon as drawn. Prove that H, M, D lie on one straight line.



Sources and classroom notes

Original contest identifiers are printed beside every question

These are faithful English classroom restatements with newly drawn figures. Multiple-choice options have been omitted; the mathematical data and requested answers have been preserved. Russian-language tasks from the supplied extract have been translated. The hints and explanatory solutions were written for this pack. **Supplied material.** “Visual geometry”, Chapter 7, printed pages 48-52, from the PDF supplied by the teacher. The worksheet labels retain section numbers and the contest credits printed in that extract. Those credits are taken from the supplied source, rather than independently checked against each original Russian contest paper. **Attribution exception.** B01 is an exercise in section 7.1.9 without a contest credit. It is labelled as a collection exercise, not as a verified olympiad question.

International sources. UKMT papers include the Junior Mathematical Challenge and Junior Mathematical Olympiad. AMC 8 is the MAA junior multiple-choice competition; Gauss is CEMC's Canadian contest. These are useful olympiad preparation sources, with different formats and difficulty levels.

UKMT Junior Mathematical Challenge 2022

https://ukmt.org.uk/wp-content/uploads/2023/08/JMC_2022_Paper.pdf

UKMT Junior Mathematical Challenge 2024

<https://ukmt.org.uk/wp-content/uploads/2024/04/JMC-2024-Question-Paper-4.pdf>

UKMT Junior Mathematical Olympiad 2023

<https://ukmt.org.uk/wp-content/uploads/2023/08/Junior-Mathematical-Olympiad-2023-Paper-2.pdf>

UKMT Junior Mathematical Olympiad 2024: official solutions and questions

<https://ukmt.org.uk/wp-content/uploads/2024/06/JMO-2024-Solutions.pdf>

MAA AMC 8, 2022: original MAA paper, archived reprint

<https://ivyleaguecenter.org/wp-content/uploads/2022/01/2022-amc-8-problems-and-answers.pdf>

MAA AMC 8, 2023: official question paper

<https://maa.org/wp-content/uploads/2024/08/2023-Problems-AMC8-PDF.pdf>

CEMC Gauss 2023, Grade 7: official paper

<https://cemc.uwaterloo.ca/sites/default/files/documents/2023/2023Gauss7Contest.html>

CEMC Gauss 2024, Grade 7: official paper

<https://cemc.uwaterloo.ca/sites/default/files/documents/2024/2024Gauss7Contest.html>

The 2022 AMC 8 link is an archived copy of the original MAA paper; this pack uses its MAA question statements and figures as the source, not third-party explanations. This selection is intended for classroom teaching.