

Visual geometry

Year 6-7 | Four-lesson problem ladder

Use a separate sheet for your working. Explain your thinking with words, calculations and sketches. Figures are schematic; use stated lengths and properties, not measurements from the page.

This pack contains 24 problems, arranged in four sets of six. Each set starts with accessible tasks and ends with a stretch task. Your teacher will choose the core questions for each meeting. Separate hints and explained solutions are available later. **Tools to keep nearby**

- **Perimeter** is the length of the outside boundary. For a rectangle, $P = 2(L + W)$.
- **Area** measures the region inside. Rectangle: $A = LW$; square: $A = s^2$; triangle: $A = \frac{1}{2}bh$, using a perpendicular height.
- On a grid, count whole cells and pair half-cells. Shared edges inside a combined shape are not part of its perimeter.
- Around a closed path, total rightward travel equals total leftward travel; total upward travel equals total downward travel.
- **Volume** of a cuboid is LWH . A cube has six square faces, so its surface area is $6s^2$. If lengths are multiplied by k , areas are multiplied by k^2 .
- A cube net folds into six faces. Opposite faces do not share an edge. Try folding a paper model.
- A triangle's third side x satisfies $|a - b| < x < a + b$. The inequalities are strict.

Three useful habits

Sketch the situation. Count by a clear rule, such as size or direction. Check that your answer meets every condition. **The four steps**

1. **Count and measure.** Counting gaps, segments and squares; turning words into a picture.
2. **Area and hidden perimeter.** Whole and half squares; overlaps; recovering missing lengths.
3. **Build and reshape.** Cube nets, scaling, conserved volume and shared edges.
4. **Explain why.** Route comparisons, corner counts, bounds and verified constructions.

Lesson 1 Count and measure

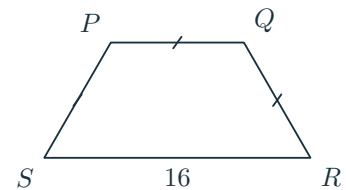
Counting gaps, segments and squares; turning words into a picture.

Suggested route: first four questions as the core; last two as extensions. Your teacher may adjust this.

A01 Three equal sides

Canada | CEMC Gauss 2023, Grade 7, Q6

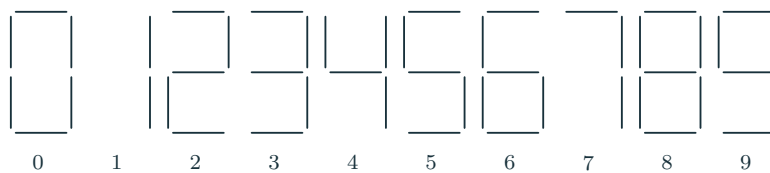
In quadrilateral $PQRS$, the sides PQ , QR and SP have equal lengths. Side SR is 16 cm and the perimeter is 40 cm. Find PQ .



A02 Metal digits

Supplied extract 7.1.1 | Lomonosov 2019, Grades 5-6, Q1

The pictured digits are made from uniform metal wire of the same thickness; all short segments have equal length. The digit 1 weighs 1 kg. How much do the four digits forming 2019 weigh together?



A03 Opposite children

Britain | UKMT Junior Mathematical Challenge 2022, Q14

Children stand at equally spaced positions around a circle, numbered consecutively $1, 2, 3, \dots$. Child 13 stands opposite child 35. How many children are there?

A04 Two painters

Supplied extract 7.1.2 | All-Russian Olympiad 2019, school round, Grade 5, Q2

A corridor is 15 m long. Both painters work towards its far end. The first starts 2 m from the beginning and paints 9 m red. The second paints 10 m yellow and stops 1 m before the far end. How many metres are painted exactly once?

A05 A rectangle from its perimeter

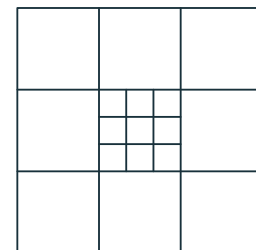
Canada | CEMC Gauss 2024, Grade 7, Q12

A rectangle is twice as long as it is wide. Its perimeter is 120 cm. What is its width?

A06 Find every square

Britain | UKMT Junior Mathematical Challenge 2022, Q9

How many squares of all sizes have their complete boundaries drawn in the diagram? The central cell of the large 3×3 grid is itself divided into a 3×3 grid.



Lesson 2 Area and hidden perimeter

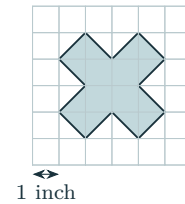
Whole and half squares; overlaps; recovering missing lengths.

Suggested route: first four questions as the core; last two as extensions. Your teacher may adjust this.

A07 The multiplication logo

USA | MAA AMC 8, 2022, Q1

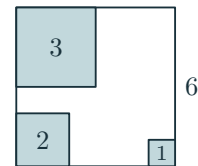
The shaded multiplication symbol lies on a grid of 1-inch squares. What is its area, in square inches?



A08 Three shaded squares

Canada | CEMC Gauss 2024, Grade 8, Q10

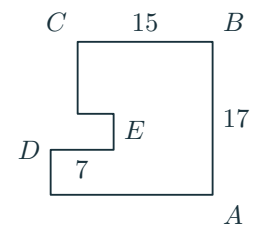
A square of side 6 contains three non-overlapping shaded squares of sides 3, 2 and 1, as shown. Find the unshaded area.



A09 A hidden perimeter

Supplied extract 7.1.5 | Belchonok 2020, Grade 5, Q4

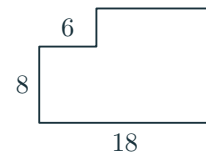
All consecutive sides of the polygon are perpendicular. Given $AB = 17$, $BC = 15$ and $DE = 7$, find its perimeter.



A10 A stepped shape

Britain | UKMT Junior Mathematical Challenge 2023, Q10

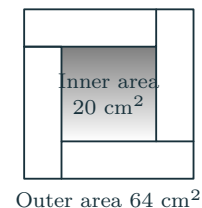
The stepped polygon has the labelled lengths, in centimetres, and perimeter 60 cm. Find its area.



A11 Four identical rectangles

Britain | UKMT Junior Mathematical Challenge 2023, Q13

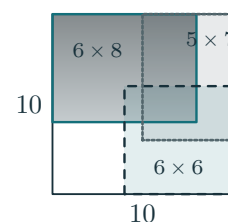
Four congruent rectangles form the frame shown. The outer square has area 64 cm^2 and the inner square has area 20 cm^2 . Find the perimeter of one rectangle.



A12 Three layers of carpet

Supplied extract 7.3.3 | All-Russian Olympiad 2022, school round, Grade 5, Q4

A hall is a $10 \times 10 \text{ m}$ square. The top-left carpet is 8 m wide and 6 m high; the bottom-right carpet is a $6 \times 6 \text{ m}$ square; the top-right carpet is 5 m wide and 7 m high, as shown. What area is covered by all three?



Lesson 3 Build and reshape

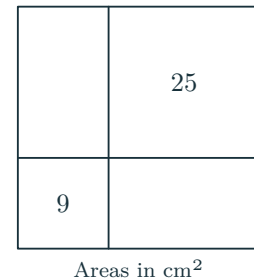
Cube nets, scaling, conserved volume and shared edges.

Suggested route: first four questions as the core; last two as extensions. Your teacher may adjust this.

A13 Two squares, two rectangles

Britain | UKMT Junior Mathematical Olympiad 2024, A3

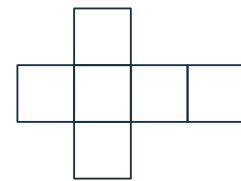
A square is partitioned into two identical rectangles and two squares of areas 9 cm^2 and 25 cm^2 , as shown. Find the area of the large square.



A14 Opposite faces

Supplied extract 7.2.1 | Belchonok 2022, Grade 5, Q1

Copy this net onto your working paper. Put 4, 8, 12, 16, 20, 24 in its six squares, once each, so that the sum on each pair of opposite cube faces is the same. Give one arrangement.



A15 Painting a larger cube

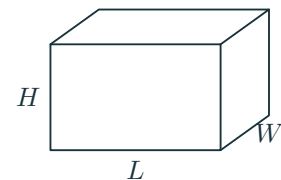
Supplied extract 7.2.3 | Nadezhda Energetiki 2021, Grade 5, Q2

Painting all faces of a cube of side 2 cm needs 2 g of paint. How much paint is needed for all faces of a cube of side 6 cm, with the same thickness of coating?

A16 A cuboid made of cubes

Britain | UKMT Junior Mathematical Olympiad 2024, A2

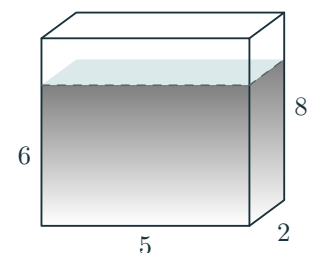
A solid cuboid is assembled from 42 cubes of side 1 cm. Its rectangular base has perimeter 16 cm. Find its height.



A17 Tipping a water container

Canada | CEMC Gauss 2023, Grade 7, Q18

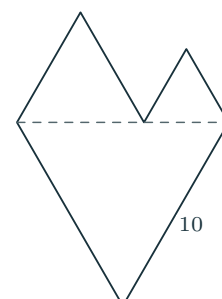
A closed cuboid has dimensions $2 \times 5 \times 8 \text{ cm}$. It stands on its 2×5 face, with water 6 cm deep. It is tipped onto its largest face. What is the new water depth?



A18 A hexagon made from triangles

Britain | UKMT Junior Mathematical Challenge 2024, Q11

Three equilateral triangles form the hexagon shown. The largest triangle has side 10 cm. Find the perimeter of the hexagon.



Lesson 4 Explain why

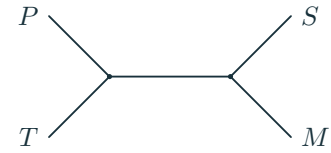
Route comparisons, corner counts, bounds and verified constructions.

Suggested route: first four questions as the core; last two as extensions. Your teacher may adjust this.

A19 Village roads

Supplied extract 7.1.4 | All-Russian Olympiad 2014, municipal round, Grade 5, Q4

The diagram shows the complete road network between villages P, T, S, M . Distances along roads are $PT = 9$, $PS = 13$, $TS = 8$ and $TM = 14$ km. Find PM .



A20 Counters on outlines

Supplied extract 7.1.6 | Lomonosov 2018, Grades 5-6 / 7-8, Q2

The same set of counters can form the outline of an equilateral triangle or of a square, with equal counts on the sides of each figure. A square side contains two fewer counters than a triangle side. Side counts include the corner counters. How many counters are there?

A21 Can these lengths form a triangle?

Supplied extract 7.1.7 | Pokori Vorobyovy Gory 2020, Grades 5-6, Q3

Two sides of a triangle are 12.67 cm and 0.55 cm. The third side has a whole-number length in centimetres. Find it.

A22 A donkey and a piece of pasta

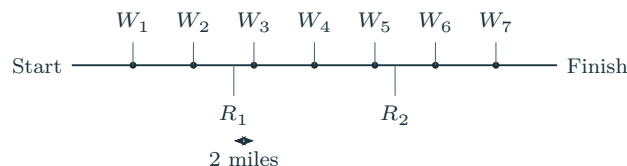
USA | MAA AMC 8, 2022, Q11

A donkey starts with one long piece of pasta. Each bite removes a 3-inch piece from the middle of one remaining piece, splitting that piece in two. At the end, 10 pieces remain, with combined length 17 inches. Find the original length.

A23 Stations along a race

USA | MAA AMC 8, 2023, Q13

Seven water stations are equally spaced between a race start and finish, dividing the course into eight equal intervals. Two repair stations divide it into three equal intervals. The third water station is 2 miles beyond the first repair station. Find the course length.



A24 Forty-nine squares

Supplied extract 7.3.1 | Formula of Unity / Third Millennium 2023, Grade 5, Q2

Use all 49 identical unit squares, without cutting or overlapping them, to make two rectangles. The perimeter of one rectangle must be twice the perimeter of the other. Give one construction and justify it.

Sources and classroom notes

Original contest identifiers are printed beside every question

These are faithful English classroom restatements with newly drawn figures. Multiple-choice options have been omitted; the mathematical data and requested answers have been preserved. Russian-language tasks from the supplied extract have been translated. The hints and explanatory solutions were written for this pack. **Supplied material.** “Visual geometry”, Chapter 7, printed pages 48-52, from the PDF supplied by the teacher. The worksheet labels retain section numbers and the contest credits printed in that extract. Those credits are taken from the supplied source, rather than independently checked against each original Russian contest paper. **International sources.** UKMT papers include the Junior Mathematical Challenge and Junior Mathematical Olympiad. AMC 8 is the MAA junior multiple-choice competition; Gauss is CEMC's Canadian contest. These are useful olympiad preparation sources, with different formats and difficulty levels.

UKMT Junior Mathematical Challenge 2022

https://ukmt.org.uk/wp-content/uploads/2023/08/JMC_2022_Paper.pdf

UKMT Junior Mathematical Challenge 2023

https://ukmt.org.uk/wp-content/uploads/2023/08/JMC-2023_Paper.pdf

UKMT Junior Mathematical Challenge 2024

<https://ukmt.org.uk/wp-content/uploads/2024/04/JMC-2024-Question-Paper-4.pdf>

UKMT Junior Mathematical Olympiad 2024: official solutions and questions

<https://ukmt.org.uk/wp-content/uploads/2024/06/JMO-2024-Solutions.pdf>

MAA AMC 8, 2022: original MAA paper, archived reprint

<https://ivyleaguecenter.org/wp-content/uploads/2022/01/2022-amc-8-problems-and-answers.pdf>

MAA AMC 8, 2023: official question paper

<https://maa.org/wp-content/uploads/2024/08/2023-Problems-AMC8-PDF.pdf>

CEMC Gauss 2023, Grade 7: official paper

<https://cemc.uwaterloo.ca/sites/default/files/documents/2023/2023Gauss7Contest.html>

CEMC Gauss 2024, Grade 7: official paper

<https://cemc.uwaterloo.ca/sites/default/files/documents/2024/2024Gauss7Contest.html>

CEMC Gauss 2024, Grade 8: official paper

<https://cemc.uwaterloo.ca/sites/default/files/documents/2024/2024Gauss8Contest.html>

The 2022 AMC 8 link is an archived copy of the original MAA paper; this pack uses its MAA question statements and figures as the source, not third-party explanations. This selection is intended for classroom teaching.