

Digits and decimal notation

Student questions | 13 problems | Work on separate paper

Give reasons for your answers. A bar denotes a number made from digits: $\overline{abc} = 100a + 10b + c$, not $a \times b \times c$. A number's first digit is nonzero. Work on separate paper; Problems 10-13 are optional extensions.

Ladder A Read the place values

1. A mind-reading trick

MSU maths circle, 22 November 2025, Problem 7.0

Vasya asks you to choose an integer from 1 to 9.

- (a) Multiply it by 11 and tell him only the last digit. He immediately knows your original number. Explain the trick.
- (b) Instead, multiply your chosen number by 11 and then by 21. You tell Vasya the result, and he again identifies your number immediately, without dividing. Explain how.

2. Four connected digits

MSU maths circle, Problem 7.2

Find the four-digit number whose first digit is one third of its third digit, whose second digit is the sum of its first and third digits, and whose fourth digit is three times its third digit.

3. Append one digit

MSU maths circle, Problem 7.8

Matvey appends the digit 7 to the right of his classroom number. Adding this new number to the original classroom number gives 4671. What is the classroom number?

Ladder B Turn digit clues into equations

4. Add a number to its reversal

USA: AMC 8, 2016, Problem 11

A two-digit number and its digit-reversal add to 132. How many two-digit numbers have this property?

5. Recover the bicycle-lock code

MSU maths circle, Problem 7.1

Petya's forgotten bicycle-lock code has two digits. Adding the sum of those digits to the code reverses its digits. Find the code.

6. A password made from two digits

MSU maths circle, Problem 7.5

Sergey's password is a seven-digit number. Its first three digits are equal, and its last four digits are equal. Its digit sum is a two-digit number whose first and last digits match the first and last digits of the password. Find the password.

7. The two middle digits

MSU maths circle, Problem 7.3

Find all four-digit numbers for which the number formed by the two middle digits is seven times the thousands digit and twice the units digit.

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Student questions | continued

8. A letter sum

MSU maths circle, Problem 7.6

Different letters stand for different digits, and equal letters for equal digits. Leading digits are nonzero. Solve

$$\begin{array}{r} A B C \\ A C \\ + \quad A \\ \hline B C C \end{array}$$

Ladder C Find structure and prove it [10-13 optional]

9. Build the longest digit chain

MSU maths circle, Problem 7.4

Every pair of neighbouring digits in a positive integer, read from left to right, forms a two-digit multiple of 23. What is the greatest possible number of digits in the integer?

10. Reverse five digits and subtract

MSU maths circle, Problem 7.7 | Extension

Take any five-digit number and subtract the number obtained by reversing its digits. Prove that the result is divisible by 11. Any leading zeros after reversal are ignored.

11. Only the final three digits matter

MSU maths circle, Problem 7.9 | Extension

The five-digit number $\overline{KASKA}$ is multiplied by 99999, and the product ends in 285. Equal letters represent equal digits; different letters represent different digits. Find $\overline{KASKA}$.

12. The smallest possible multiple

UKMT Junior Mathematical Olympiad, 2024, Section B, Problem 1 | Extension

Using only the digits 0 and 1, form the smallest positive integer divisible by 36. Prove that your number is the smallest.

13. One hundred ones

MSU maths circle, Problem 7.10 | Final stretch

The number 2999 is multiplied by the number consisting of 100 ones. Find the sum of the digits of the product.